

## Turbulence Modeling

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### 1. Motivation and Objectives

- (1) Examine the performance of existing two-equation eddy viscosity models and develop better models for the near-wall turbulence using direct numerical simulations of plane channel and boundary layer flows.
- (2) Use the asymptotic near-wall behavior of turbulence to examine the problems of current second-order closure models and develop new models with the correct near-wall behavior.
- (3) Use Rapid Distortion Theory to analytically study the effects of mean deformation (especially due to pure rotation) on turbulence, obtain analytical solutions for the spectrum tensor, Reynolds stress tensor, anisotropy tensor and its invariants, which can be used in the turbulence model development.
- (4) Explore the potential of the renormalization group (RNG) theory in turbulence modeling.
- (5) Modeling of compressible turbulent flows.
- (6) Modeling of bypass transition.

### 2. Work Accomplished and Ongoing Work

#### 2.1 $k$ - $\epsilon$ model

The  $k$ - $\epsilon$  model is still the most widely used model for computing engineering flows. We have examined the near-wall behavior of various eddy viscosity models proposed by different researchers, and have studied the near-wall behavior of the terms in the  $k$ -equation budget. We found that the modeled eddy viscosity in many existing  $k$ - $\epsilon$  models does not possess correct near-wall behavior and the pressure transport term in the  $k$ -equation is not modeled appropriately. Based on the near-wall asymptotic behavior of the eddy viscosity and the pressure transport term in the  $k$ -equation, a new set of improved closure models has been obtained. In addition, a modeled equation for the dissipation rate is derived more rationally. This work is reported in NASA TM 103221 ICOMP-90-16<sup>[1]</sup>.

In addition, all the existing two-equation models (except Jones & Laun-

der model, which unfortunately does not work well even for some simple flows) have an “unacceptable” wall distance parameter ( $y^+$ ) in their eddy viscosity damping function  $f_\mu(y^+)$ . This will result in an unphysical zero eddy viscosity near the separation region. In addition,  $y^+$  can not be well defined in many flows with complex geometry. To remove this deficiency, Dr. V. Michelassi, Dr. A. Hsu and I proposed two new eddy viscosity damping functions, and both of them are independent of the wall distance. The new models have been satisfactorily tested in channel and boundary layer flows. This work is reported in two papers: AIAA-91-0611<sup>[2]</sup> and NASA TM/ICOMP/CMOTT<sup>[3]</sup>.

## 2.2 Second order modeling of near-wall turbulence

The main emphasis is on developing a near-wall turbulence model for the velocity pressure gradient correlation and the dissipation tensor in the Reynolds-stress equation. A modeled dissipation rate equation is also derived more rationally. Near a wall, a reduction in velocity fluctuations normal to the wall becomes significant. Because of this wall effect, the viscous diffusion term in the Reynolds-stress equations becomes the leading term and it must be properly balanced by the other terms. We have used this as a model constraint for developing a model for the pressure and dissipation terms. To test the models, a fully developed channel flow and boundary layer flows are chosen as the test flows, for which direct numerical simulations and experimental data are available for comparison. The modeled Reynolds stress equations for the channel flow are steady one-dimensional, and for boundary layer flows are steady two-dimensional. Therefore model testing will be very accurate. This part of work<sup>[4]</sup> is reported in the paper: *Proceedings of the International Symposium on Engineering Turbulence Modeling and Measurements* and NASA TM 103222 ICOMP-90-0017.

## 2.3 Second order modeling of a three-dimensional boundary layer

A study of three-dimensional effects on turbulent boundary layer was achieved by the direct numerical simulation of a fully developed turbulent channel flow subjected to transverse pressure gradient (see *Physics of Fluids*, Vol.2 N0.10, 1990, pp. 1846-1853). The time evolution of the flow was studied. The results show that, in agreement with experimental data, the Reynolds stresses are reduced with increasing three-dimensionality and that, near the wall, a lag develops between the stress and the strain rate. In addition, we found that the turbulent kinetic energy also decreased. To model these three-dimensional effects on the turbulence, we have tried different two-equation models and second order closure models. None of the current

closure models can predict the reductions in the shear stress and turbulent kinetic energy observed in direct numerical simulations. Detailed studies of the Reynolds-stresses budgets were carried out. One of the preliminary conclusions from these budget studies is that the velocity pressure-gradient term in the normal stress equation  $\langle v^2 \rangle$  plays a dominant role in the reduction of shear stress and kinetic energy. These budgets have been used to guide the development of better models for three dimensional turbulent boundary layer flows. This work<sup>[5]</sup> was presented in the American Physical Society Forty-Third Annual Meeting, November, 1990.

## 2.4 The effect of rotation on turbulence

In addition to the above studies of second order closure models, we have carried out some RDT analysis on simple homogeneous turbulent flows. An order of magnitude analysis shows that under the condition of  $S\langle q^2 \rangle/\epsilon \gg \sqrt{R_t}$ , the equations for turbulent velocity fluctuations can be approximated by a linear set of equations, and if  $S\langle q^2 \rangle/\epsilon \gg R_t^{3/4}$ , then the turbulent velocity equations can be further approximated by an inviscid linear equation. Therefore, RDT can be used to analytically study some very basic turbulent flows, such as, homogeneous shear flows, irrotational strain flows and pure rotational flows. This work focuses on the effect of rapid rotation on turbulence using RDT. We have obtained analytical expressions for velocity, the spectrum tensor, Reynolds-stress, the anisotropy tensor and its invariants. The solutions show that the turbulence is strongly affected by the rapid rotation. Using RDT, we can calculate the rapid pressure-stain term exactly and we can obtain very useful information for developing corresponding turbulence models. See the report<sup>[6]</sup> for this work.

## 2.5 Renormalization Group Theory (RNG) in turbulence modeling

RNG method has been introduced to the turbulence modeling mainly in the Large Eddy Simulation (LES) of turbulence with a subgrid scale model. One also attempted to use it to develop Reynolds-averaged turbulence model equations, for example,  $k-\epsilon$  model equations. However, we found that there are a few fundamental concepts and important procedures used in the derivation of those model equations which are not clear and well justified. Dr. Z. Yang and I are working on this subject and try to explore the potential of RNG in the turbulence modeling.

## 2.6 Modeling of compressible turbulent flows

The turbulence models for compressible flows are of great interest in hypersonic flows and turbulent combustion. The modeling scheme greatly de-

depends on the averaging schemes (i.e., conventional average, density weighted average and mixed average) used in the turbulence equations. We start with the analysis of the turbulent equations derived from the different averaging schemes to see what kind of averaging scheme is most convenient for both turbulent modeling and applications in CFD. We concentrate on the second order closure model (i.e. Reynolds stress model) and two-equation model. Dr. W. Liou and I are working on this subject. See Reference<sup>[7]</sup> for the first report on averaging schemes for compressible flows.

## 2.7 Modeling of bypass transition

Most common transition phenomena occurred in engineering flows are bypass transition. A few papers on modeling of transition with turbulence models show that the bypass transition can possibly be modeled with the modified turbulence models developed solely for turbulent flows. However, most of the work in this direction was based on the parabolic two-equation models. We expect that the bypass transition phenomena will be more appropriately described by the elliptical equations. Then, the prediction of normal stresses becomes important. Because of the inability of modeling normal stresses with the two-equation models, we are pursuing the elliptical Reynolds stress model equations for the bypass transition studies. Dr. Z. Yang and I are working on the improvement of our previous near-wall Reynolds stress model for the purpose of modeling bypass transition.

## 2.8 Modeling of scalar turbulence:

Modeling of scalar turbulence is of great importance in turbulent heat transfer. Eddy viscosity models often fail in the prediction of heat transfer in many shear flows. We have developed a set of second order closure models based on the joint realizability (between velocity and scalar) and the experiments. Dr. A. Shabbir and I are working on this subject. A paper<sup>[8]</sup> was presented in *the Lumley Symposium: Recent developments in turbulence, November, 1990*.

## 3. Publications:

1. Shih, T.-H., 1990, "An Improved  $k$ - $\epsilon$  Model for Near-Wall Turbulence and Comparison with Direct Numerical Simulation," NASA TM 103221 ICOMP-90-16.
2. Shih, T.-H. and Hsu, A.T., 1991, "An Improved  $k$ - $\epsilon$  Model for Near-Wall Turbulence," AIAA-91-0611.

3. Michelassi, V. and Shih, T.-H., 1991, "Low Reynolds Number Two-Equation Modeling of Turbulent Flows," NASA TM 104368, ICOMP-91-06, CMOTT-91-01.
4. Shih, T.-H. and Mansour, N.N., 1990, "Modeling of Near-Wall Turbulence," *Proceeding of the International Symposium on Engineering Turbulence Modeling and Measurements*, September, 1990, Dubrovnic, Yugoslavia, Editors: W. Rodi, E.N. Ganic. or, NASA TM 103222 ICOMP-90-0017.
5. Shih, T.-H., 1990, "Modeling of 3D Turbulent Boundary Layer Flows," American Physical Society Forty-Third Annual Meeting, 1990, Cornell, U. Ithaca, New York.
6. Shih, T.-H., 1991, "Rapid Distortion Theory on Homogenous Turbulence with Rapid Rotation," CMOTT Report.
7. Liou, W.W. and Shih, T.-H., 1991, "On the Basic Equations for the Second-order Modeling of Compressible Turbulence," CMOTT-91-06.
8. Shih, T.-H. and Shabbir, A., 1990, "Advances in Modeling the Pressure Correlation Terms in the Second Order Moment Equations," *the Lumley Symposium: Recent developments in turbulence, November, 1990, ICASE, NASA Langley Research Center, Edited by T.B. Gatski, S.Sarkar and C.G. Speziale*
9. Shih, T.-H., 1990, "Advancements in Engineering Turbulence Modeling," 9th NASP Technology Symposium, Paper-105, November, 1990, Orlando FL.
10. Shih, T.-H., Chen, J.-Y. and Lumley, J.L., 1991, "Second Order Modeling of Boundary Free Turbulent Shear Flows," AIAA 91-1779.